SELF-OSCILLATION DURING A PHASE TRANSITION IN THERMOPHYSICAL AUTOMATIC CONTROL SYSTEMS

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Algorithms and the results of analyzing the equation of an automatic control system with a phase transition in the thermal feedback element are presented.

The problem of self-oscillations in automatic control systems with thermal feedback was first studied during the development of the theory for a new method of measuring thermophysical characteristics. It was shown in [1] that for a certain feedback level self-oscillations are excited in this kind of system, and a temperature wave propagates in the same feedback element with angular frequency $\omega(k)$. As follows from [1], for a system with proportional control, the wave number is constant and the temperature conductivity can be found from the dispersion relation

$$a = \omega / 2k^2 \tag{1}$$

from a single experimentally determined quantity, namely, the self-oscillation frequency. The question of the regime of excitation of self-oscillations and the correction to the frequency of a nonlinear systems was examined in detail in [2], where one of the small parameter methods [3, 4] was used to analyze the equation of a nonlinear automatic control system.

Although numerous problems arose as mathematical models of systems that realize the self-oscillation method, the range of their applicability is significantly broader. Equations similar to those examined in [1, 2] describe heat control systems, in which the source of heat and the heat-sensitive element are spatially separated. The feedback in these systems is accomplished through a perturbation of the temperature field of the medium, in which the heat source and the temperature sensor are situated. If phase transformations are possible in the medium, then the change in thermophysical characteristics induced by them, and consequently, the level of feedback will lead to a change in the operating regime of the automatic control system. Under certain conditions, this can cause the system to lose stability or become less sensitive. To study the behavior of such systems, we require the method for analyzing automatic control systems with thermal feedback developed in [1, 2] extended to systems in which the feedback element is found in a biphase state. Below, an algorithm for this type of analysis is discussed for an example of an automatic control system with proportional control and uniform heat flow in the feedback element.

We shall examine an automatic control system (Fig. 1) consisting of a regulator 1, planar heater 2, differential thermocouple 3, and voltage reference 4. Let the heater and one of the thermocouple junctions be situated in the thermally conducting medium 5, immersing the feedback pickup loop, while the thermostat 6 serves to discharge the liberated heat. The

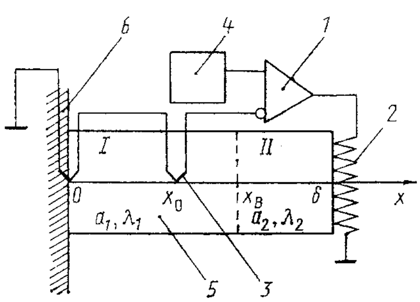

Fig. 1. Automatic control system with a proportional regulator and thermal feedback element in biphase state: x_B is the phase boundary; a_1, λ_1, a_2, λ_2 are the thermal diffusivity and thermal conductivity in phases I and II.

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temperature field of the medium can be conveniently described in terms of the deviations from the thermostat temperature, applied to its second thermocouple junction.

We shall derive the equation for the automatic control system, having eliminated the equations forming the system's elements:

$$
\begin{aligned}
u_1 &= \alpha T(x_0,\ t) \text{ — thermocouples}\\
u_2 &= K(u_0 - u_1)\,\sigma(u_0 - u_1) \text{ — controller}\\
P &= u_2^2/R \text{ — heater}\\
\lambda_2 T_2'(\delta,\ t) &= P/S \text{ — sample}
\end{aligned}
\tag{2}
$$

and the intermediate variables u_1, u_2, and P:

$$
\lambda_2 T_2'(\delta,\ t) = \frac{K^2\alpha^2}{SR}\left[\frac{u_0}{\alpha} - T_1(x_0,\ t)\right]^2 \sigma\left[\frac{u_0}{\alpha} - T_1(x_0,\ t)\right]. \tag{3}
$$

In order to close the system of equations (2), we shall add to Eq. (3) the equation for the thermal conductivity for each phase

$$
\dot{T}_1(x,\ t) = a_1 T_1''(x,\ t);\ \dot{T}_2(x,\ t) = a_2 T_2''(x,\ t) \tag{4}
$$

with the corresponding boundary conditions

$$
T_1(0,\ t) = 0;\ \lambda_2 T_2'(x_B,\ t) = \lambda_1 T_1'(x_B,\ t) + \rho q \dot{x}_B. \tag{5}
$$

The last of these equations is called the Stephan condition and, as does Eq. (3), expresses conservation of energy. In final form, the equation for the automatic control system is the nonlinear thermal conductivity boundary value problem

$$
\begin{gathered}
\dot{T}_1(x,\ t) = a_1 T_1''(x,\ t);\quad \dot{T}_2(x,\ t) = a_2 T_2''(x,\ t),\\
T_1(0,\ t) = 0;\ T_1(x_B,\ t) = T_2(x_B,\ t) = T_0,\\
\lambda_2 T_2'(x_B,\ t) = \lambda_1 T_1'(x_B,\ t) + \rho q \dot{x}_B,\\
T_2'(\delta,\ t) = \varkappa\left[\frac{u_0}{\alpha} - T_1(x_0,\ t)\right]^2 \sigma\left[\frac{u_0}{\alpha} - T_1(x_0,\ t)\right] \text{ for } \bar{x}_B > x_0,\\
T_2'(\delta,\ t) = \varkappa\left[\frac{u_0}{\alpha} - T_2(x_0,\ t)\right]^2 \sigma\left[\frac{u_0}{\alpha} - T_2(x_0,\ t)\right] \text{ for } \bar{x}_B < x_0,
\end{gathered}
\tag{6}
$$

better known as the biphase single-front Stephan problem [5]. It seems reasonable that the phase state of the medium varies only under the influence of the external source, while the thermal conductivity coefficients λ_1, λ_2 and the thermal diffusivities a_1 and a_2 are assumed constant and equal in each of the phases.

To study the solution of system (6), we chose as the small parameter $\varepsilon = (\kappa - \kappa_c)/\kappa_c$, which is the relative deviation of the coefficient $\kappa = K^2\alpha^2/\lambda_2 SR$ from its critical value κ_c at which self-oscillations arise in the system. We shall expand the parameter ε in powers of the auxiliary parameter ξ, which represents the amplitude of the first harmonic:

$$
\varepsilon = b_2\xi^2 + b_4\xi^4 + \dots \tag{7}
$$

We shall also define

$$
\begin{gathered}
t = (1 + c)\tau;\ c = c_2\xi^2 + c_4\xi^4 + \dots,\\
T_1(x,\ \tau) = \overline{T}_1 + \xi\Theta_1(x,\ \tau) + \xi^2\Theta_2(x,\ \tau) + \dots,\\
T_2(x,\ \tau) = \overline{T}_2 + \xi\Xi_1(x,\ \tau) + \xi^2\Xi_2(x,\ \tau) + \dots,\\
x_B(\tau) = \bar{x}_B + \tilde{x}_B(\tau) = \bar{x}_B + \xi x_1(\tau) + \xi^2 x_2(\tau) + \dots
\end{gathered}
\tag{8}
$$

Substituting Eqs. (7) and (8) into Eqs. (6) and grouping terms of the same power of ξ, we obtain a recurrence sequence of linear inhomogeneous boundary value problems. Expressions of the form

$$T_j^{(n)}\left(\bar{x}_B + \tilde{x}_B(\tau)\right),\ j = 1,\ 2;\ n = 0,\ 1, \tag{9}$$

included in the matching condition and the Stephan condition, must be expanded in a Taylor power series $\tilde{x}_B(\tau) = \xi x_1(\tau) + \xi^2 x_2(\tau) + \ldots$ and it is necessary to group terms taking this expansion into account.

In view of the complexity of the problem, we shall confine ourselves to linear analysis, examining the first two problems of the sequence:

$$\begin{gathered}
\bar{T}_1''(x) = 0;\ \bar{T}_2''(x) = 0,\\
\bar{T}_1(0) = 0;\ \bar{T}_1(\bar{x}_B) = \bar{T}_2(\bar{x}_B) = T_0,\\
\lambda_2 \bar{T}_2'(\bar{x}_B) = \lambda_1 \bar{T}_1'(\bar{x}_B),\\
\bar{T}_2'(\delta) = \varkappa_c \left[\frac{u_0}{\alpha} - \bar{T}_1(x_0)\right]^2 \sigma \left[\frac{u_0}{\alpha} - \bar{T}_1(x_0)\right],\ \bar{x}_B > x_0,\\
\bar{T}_2'(\delta) = \varkappa_c \left[\frac{u_0}{\alpha} - \bar{T}_2(x_0)\right]^2 \sigma \left[\frac{u_0}{\alpha} - \bar{T}_2(x_0)\right],\ \bar{x}_B < x_0,
\end{gathered} \tag{10}$$

$$\begin{gathered}
\dot{\Theta}_1(x,\ \tau) = a_1 \Theta_1''(x,\ \tau);\ \dot{\Xi}_1(x,\ \tau) = a_2 \Xi_1''(x,\ \tau),\\
\bar{T}_1'(\bar{x}_B)\, x_1(\tau) + \Theta_1(\bar{x}_B,\ \tau) = 0,\\
\bar{T}_2'(\bar{x}_B)\, x_1(\tau) + \Xi_1(\bar{x}_B,\ \tau) = 0,\\
\lambda_2 \Xi_1'(\bar{x}_B,\ \tau) = \lambda_1 \Theta_1'(\bar{x}_B,\ \tau) + \rho q \dot{x}_1(\tau);\ \Theta_1(0,\ \tau) = 0,\\
\Xi_1'(\delta,\ \tau) = \frac{A_{c1}}{x_0} \Theta_1(x_0,\ \tau),\ \bar{x}_B > x_0,\\
\Xi_1'(\delta,\ \tau) = \frac{A_{c2}}{x_0} \Xi_1(x_0,\ \tau),\ \bar{x}_B < x_0,\\
\text{где}\ A_{cj} = 2\varkappa_c x_0 [u_0/\alpha - \bar{T}_j(x_0)].
\end{gathered} \tag{11}$$

The dimensionless parameter A is the product of the coefficients of the transformation of the constant signal of all elements of a closed automatic control system, and shall be called a generalized gain coefficient. The electrical K, u_0, $\alpha(T)$, $R(T)$, thermophysical $\lambda_1(T)$, $\lambda_2(T)$, and less commonly geometric x_0, δ, S parameters entering into the system in the process of operation can vary with temperature or can be controlled. If the modulus of A exceeds some critical value A_c, then self-oscillations arise in the system. The parameter, the variation of which leads to the loss of stability, is called the bifurcation parameter. The goal of further analysis of Eqs. (10) and (11) consists of deriving expressions for the temperature field and equations for A_c and ω.

The solution of the stationary problem (10):

$$\bar{T}_1(x) = Bx;\ \bar{T}_2(x) = \frac{\lambda_1}{\lambda_2} B (x - \bar{x}_B) + T_0, \tag{12}$$

$$B = \frac{u_0}{\alpha x_0}\left(D - \sqrt{D^2 - 1}\right);\ D = 1 + \lambda_1 SR/2\alpha u_0 x_0 K^2 \text{ for } \bar{x}_B > x_0;$$

$$\begin{gathered}
\bar{T}_1(x) = B(\bar{x}_B)\, x;\ \bar{T}_2(x) = \frac{\lambda_1}{\lambda_2} B(\bar{x}_B)(x - \bar{x}_B) + T_0,\\
B(\bar{x}_B) = u_0 \left[D(\bar{x}_B) - \sqrt{D^2(\bar{x}_B) - 1}\right] / \alpha [\lambda_1 (x_0 - \bar{x}_B)/\lambda_2 + \bar{x}_B];
\end{gathered} \tag{13}$$

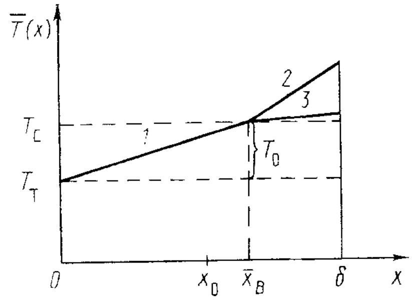

Fig. 2. Stationary temperature of the thermal feedback element: 1) $\bar{T}_1(x) + T_T$; 2) $\bar{T}_2(x) + T_T$ for $\lambda_1/\lambda_2 > 1$; 3) $\bar{T}_2(x) + T_T$ for $\lambda_1/\lambda_2 < 1$.

$$D(\bar{x}_B) = 1 + \frac{\lambda_1 SR}{2\alpha u_0 K^2 \left[\frac{\lambda_1}{\lambda_2}(x_0 - \bar{x}_B) + \bar{x}_B\right]} \quad \text{for } \bar{x}_B < x_0$$

permits us to track the variation of the stationary temperature during a phase transition. The phase boundary, first located at the plane $x = \delta$, with increasing thermostat temperature is shifted to a new coordinate (Fig. 2). If none of the parameters entering into A vary, then the temperature gradient in both phases remains constant. After this, the phase boundary intersects x_0, and its further motion is related to the variation of the stationary temperature in both phases.

We seek a periodic solution to Eq. (11) in the form

$$\Theta_1(x, \tau) = V(x)\exp(i\omega\tau); \quad \Xi_1(x, \tau) = W(x)\exp(i\omega\tau). \tag{14}$$

Substituting Eq. (14) into Eq. (11) and equating the amplitudes of the oscillations of the interphase boundary in the expressions

$$x_1(\tau) = -\frac{V(\bar{x}_B)}{B}\exp(i\omega\tau); \quad x_1(\tau) = -\frac{\lambda_2}{\lambda_1}\frac{W(\bar{x}_B)}{B}\exp(i\omega\tau), \tag{15}$$

we obtain the problem for the spatial part of the oscillating components of the temperature

$$\begin{gathered} V''(x) = \frac{i\omega}{a_1}V(x); \quad W''(x) = \frac{i\omega}{a_2}W(x), \\ \lambda_1 V(\bar{x}_B) = \lambda_2 W(\bar{x}_B); \quad V(0) = 0, \\ \lambda_2 W'(\bar{x}_B) = \lambda_1 V'(\bar{x}_B) - \frac{i\omega\rho q}{B}V(\bar{x}_B), \\ W'(\delta) = \frac{A_{c1}}{x_0}V(x_0), \quad \bar{x}_B > x_0, \\ W'(\delta) = \frac{A_{c2}}{x_0}W(x_0), \quad \bar{x}_B < x_0. \end{gathered} \tag{16}$$

Its solution

$$\begin{gathered} V(x) = C\,\mathrm{sh}\,\mu_1 x; \quad W(x) = C\frac{\lambda_1}{\lambda_1}\mathrm{sh}\,\mu_1\bar{x}_B\,\mathrm{ch}\,\mu_2(x - \\ -\bar{x}_B) + C\left[\frac{\lambda_1\mu_1}{\lambda_2\mu_2}\mathrm{ch}\,\mu_1\bar{x}_B - \frac{i\omega\rho q}{\lambda_2\mu_2 B}\mathrm{sh}\,\mu_1\bar{x}_B\right]\mathrm{sh}\,\mu_2 \times \\ \times (x - \bar{x}_B); \quad \mu_j = (1+i)k_j, \quad C = 1K, \end{gathered} \tag{17}$$

contains unknown wave numbers k_1 and k_2, which can be determined from the last boundary condition of Eqs. (16). Substituting Eqs. (17) into Eqs. (16) leads to the four conditions for the critical values of A_{ci} of the generalized gain coefficient A: two for $\bar{x}_B > x_0$ and two for $\bar{x}_B < x_0$. Eliminating A_{c1} from the first pair of conditions gives

$$[l_1(\operatorname{ch}\varphi_1\cos\varphi_1-\operatorname{sh}\varphi_1\sin\varphi_1)+l_2(\operatorname{ch}\psi_1\cos\psi_1-\operatorname{sh}\psi_1\sin\psi_1)+$$
$$+\omega\rho q(\operatorname{ch}\varphi_1\sin\varphi_1+\operatorname{ch}\psi_1\sin\psi_1)/\lambda_2 B]\times$$
$$\times[l_1(\operatorname{ch}\varphi_1\cos\varphi_1+\operatorname{sh}\varphi_1\sin\varphi_1)+l_2(\operatorname{ch}\psi_1\cos\psi_1\operatorname{sh}\psi_1\sin\psi_1)-$$
$$-\omega\rho q(\operatorname{sh}\varphi_1\cos\varphi_1+\operatorname{sh}\psi_1\cos\psi_1)/\lambda_2 B]^{-1}=\operatorname{th}k_1x_0\operatorname{ctg}k_1x_0,$$
$$\varphi_1=(k_1-k_2)\bar{x}_B+k_2\delta;\ \ \psi_1=(k_1+k_2)\bar{x}_B-k_2\delta;$$
$$l_1=\lambda_1(k_1+k_2)/\lambda_2;\ \ l_2=\lambda_1(k_1-k_2)/\lambda_2, \tag{18}$$

and eliminating A_{c2} from the second gives

$$[l_1(\operatorname{ch}\varphi_1\cos\varphi_1-\operatorname{sh}\varphi_1\sin\varphi_1)+l_2(\operatorname{ch}\psi_1\cos\psi_1-\operatorname{sh}\psi_1\sin\psi_1)+$$
$$+\omega\rho q(\operatorname{ch}\varphi_1\sin\varphi_1+\operatorname{ch}\psi_1\sin\psi_1)/\lambda_2 B(\bar{x}_B)]\times$$
$$\times[l_1(\operatorname{ch}\varphi_1\cos\varphi_1+\operatorname{sh}\varphi_1\sin\varphi_1)+l_2(\operatorname{ch}\psi_1\cos\psi_1+\operatorname{sh}\psi_1\sin\psi_1)-$$
$$-\omega\rho q(\operatorname{sh}\varphi_1\cos\varphi_1+\operatorname{sh}\psi_1\cos\psi_1)/\lambda_2 B(\bar{x}_B)]^{-1}=$$
$$=[l_1\operatorname{sh}\varphi_2\cos\varphi_2-l_2\operatorname{sh}\psi_2\cos\psi_2-\omega\rho q(\operatorname{ch}\varphi_2\cos\varphi_2-$$
$$-\operatorname{ch}\psi_2\cos\psi_2-\operatorname{sh}\varphi_2\sin\varphi_2+\operatorname{sh}\psi_2\sin\psi_2)/2\lambda_2 B(\bar{x}_B)]\times$$
$$\times[l_1\operatorname{ch}\varphi_2\sin\varphi_2-l_2\operatorname{ch}\psi_2\sin\psi_2-\omega\rho q(\operatorname{sh}\varphi_2\sin\varphi_2-$$
$$-\operatorname{sh}\psi_2\sin\psi_2+\operatorname{ch}\varphi_2\cos\varphi_2-\operatorname{ch}\psi_2\cos\psi_2)/2\lambda_2 B(\bar{x}_B)]^{-1}, \tag{19}$$
$$\varphi_2=(k_1-k_2)\bar{x}_B+k_2x_0;\ \psi_2=(k_1+k_2)\bar{x}_B-k_2x_0.$$

For specific a_j, λ_j, q, and T_c, Eqs. (18) and (19) determine the spectrum of self-oscillation frequencies, i.e., the frequencies at which the phase shift of the oscillations in the complete feedback loop for a given x_B is $2\pi m$, m = 0, 1, ... Each m corresponds to a certain value of the generalized gain coefficient $A_{cj}{}^m$ for an inverting controller $A_{cj}{}^m <$ 0. Since the coefficient A varies from 0 to $-\infty$ as the control parameters u_0 and K vary from 0 to ∞, the frequency is excited first that corresponds to the maximum critical value of A. Analysis of the analogous equation for a single-phase system shows that this takes place for m = 1, i.e., $A_{cj}{}^1 > A_{cj}^{m+2}$. It might be expected that this would also be valid for biphase systems, but a specific answer can only be obtained by calculating k_1 and k_2 and substituting them into the condition for A_c. In conclusion of the linear analysis, we point out that within the adopted assumptions, Eqs. (18) and (19) permit one to calculate the critical parameters and the oscillation frequency. For a complete description of the temperature field, it is necessary to determine the amplitude of the oscillating part of the temperature ξ, which, according to the algorithm, is calculated in the fourth stage of the analysis from the condition of solvability of the fourth problem of the recurrence sequence.

As was already discussed, two approaches are possible in interpreting the results: 1) propagation of the self-oscillation method to the phase transition region; 2) analysis of the operating regime of the thermal control system. In the framework of the first approach, we indicate the main possibility for determining the complex of thermophysical characteristics from the results of a single experiment. Actually, one of the achievements of the method of self-oscillations is the fact that the measurements can be conducted in the monotonic heating regime, continuously recording the frequency of the self-oscillations. The frequency and the critical value of the coefficient A are determined by the equations derived in [1]. If the studied temperature range contains T_c, then the second phase, initially at the point x = δ, approaches the plane x = x_0 as T_T increases. But now the frequency of the self-oscillations is determined by condition (18) and varies as $\bar{x}_B$ decreases. It is significant that as $\bar{x}_B > x_0$ the same parameter $A_1 = \lambda_1\{1-\sqrt{(D+1)/(D-1)}\}/\lambda_2$ does not change, whereas its critical value A_{c1} varies. After the phase boundary is displaced into the region $0 \le \bar{x}_B \le x_0$, not only the coefficient $A_2 = \lambda_1\{1-\sqrt{[D(\bar{x}_B)+1)]/[D(\bar{x}_B)-1]}\}/\lambda_2$ vaies, but A_{c2} as well. When $\bar{x}_B = x_0$, the critical value of the gain coefficient A experiences a discontinuity:

$$A_{c1}=(\lambda_1/\lambda_2)A_{c2}. \tag{20}$$

when $T_T = T_c$, when the phase boundary is located at the plane x = 0 (end of the phase transition), condition (19) is transformed into

$$\frac{\operatorname{ch} k_2\delta \cos k_2\delta - \operatorname{sh} k_2\delta \sin k_2\delta}{\operatorname{ch} k_2\delta \cos k_2\delta + \operatorname{sh} k_2\delta \sin k_2\delta} = \operatorname{th} k_2 x_0 \operatorname{ctg} k_2 x_0, \tag{21}$$

corresponding with that derived in [1], while A_c takes on the same value that it had before the transition. The discontinuity in the critical value of the generalized gain coefficient, appearing in the form of a collapse or sharp rise in the oscillation amplitude, permits one to precisely determine the transition temperature T_c. Taking the values of the thermophysical characteristics obtained immediately before and after the transition for $a_1(T_c)$, $a_2(T_c)$, $\lambda_1(T_c)$, and $\lambda_2(T_c)$, the specific latent heat of the transition can be found from Eqs. (18) and (19).

Conditions (18) and (19) are more general than Eq. (21) and reduce to Eq. (21) when $k_1 = k_2$, $q = 0$ or when $\bar{x}_B = 0$, which corresponds to the beginning and the end of the phase transition. As a special case, from these equations follow conditions for a second order phase transition $q = 0$ and a double-layer system $\bar{x}_B = \text{const}$, $q = 0$.

If the automatic control system being examined is a thermal control system, then the obtained results can be used to calculate its critical parameters. For example, a thermostatic control system with proportional control has the best characteristics when $A = A_c + \varepsilon$, when A somewhat exceeds the critical value. The error signal u_2 here is minimized, the accuracy of thermostatic control is maximized, and the system is stable. If a phase transition takes place, then the simultaneous change of A and A_c during the motion of the phase boundary will lead either to a decrease in sensitivity ($A - A_c > \varepsilon$) or to loss of stability, in particular, to the excitation of self-oscillations ($A - A_c < 0$). Equations (18) and (19) along with the conditions for A (not cited here) enable one to calculate the critical values of the bifurcation parameters. The controlled parameters u_0 and K should be chosen such that the condition $A > A_c$ is satisfied during and after the phase transition.

As an example, we shall examine the behavior of an automatic control system with a metallic thermal feedback element near a first-order phase transition. Let the thermocouple be located in the center $x_0 = \delta/2$ of a lead cylinder of length $\delta = 0.1$ m, adiabatic conditions are maintained on the surface of which, while the heater and thermostat are positioned as shown in Fig. 1. We shall make use of the thermophysical characteristics and density of lead cited in [6]: $a_1 = 20.1\cdot10^{-6}$ m²/sec, $a_2 = 9.9\cdot10^{-6}$ m²/sec, $\lambda_1 = 31.6$ W/(m·K), $\lambda_2 = 15.5 \times$ W/(m·K), $T_c = 606.652$ K, $\rho = 11.058$ kg/m³, $q = 23.03\cdot10^3$ J/kg. The self-oscillation frequency at maximum temperature in the solid phase, calculated from the formula $\omega = 2a_1\nu^2/\delta^2$, where $\nu = 4.694105$ is the root of Eq. (21), and is equal to $8.86\cdot10^{-2}$ sec⁻¹, and $A_c = -34.6415$. These values correspond to a value of $D_c = 1.001575$, which when substituted into Eq. (12) gives the condition for the critical values of the control parameters $SR/\alpha u_0K^2 = 4.98\cdot10^{-6}$. The phase shift of the signal in the thermal feedback element is distributed as follows: approximately $3\pi/4$, more precisely $\nu/2$ ($\nu/2 \neq 3\pi/4$ due to superposition of the temperature wave reflected from the thermostat), comes from the phase difference between the temperature oscillations at points $x = x_0$ and $x = \delta$. The remaining $\pi - \nu/2$ is the phase difference of the temperature and power oscillations at the plane $x = \delta$. Now let the phase boundary be located in the middle between the thermocouple and the heater at the point $x = 3\delta/4$ and $T_T = 606$ K. Under these conditions, Eq. (18) gives $\omega = 0.134$ sec⁻¹, i.e., greater than for a double-layer system with the same parameters. This is because an additional phase shift in the feedback signal takes place at the phase boundary, which corresponds to the term with q in Eq. (18); q plays the dominant role in this case and is missing in the analogous equation for a double-layer system. The critical value of the generalized gain coefficient A_{c1} is $-6.755\cdot10^5$, an almost unallowable value, which means the absence of self-oscillations, at least for a given x_B. This result has a simple explanation: Since the feedback signal from phases II to phase I is conveyed through an oscillation in the phase boundary, then for it to shift with the required speed, determined by the self-oscillation frequency, a large heat flow density is required (especially with a small gradient in the stationary temperature) and, consequently, a large gain coefficient. If the phase boundary is located between the thermostat and the thermocouple such that $\bar{x}_B = \delta/4$, while the stationary temperature gradient of phase I remains unchanged, then the self-oscillation frequency, satisfying Eq. (19), is equal to 0.343 sec⁻¹, and the critical value of the generalized gain coefficient $A_{c2} = -0.183$. The range of variation of the controlled parameters, corresponding to the stable state of the system and determined by Eq. (13) and the condition $D > 6.01$, turns out to be significantly narrower: $SR/\alpha u_0K^2 > 1.20\cdot10^{-2}$. As $x_B \to x_0$, the critical values of A_{c1} and A_{c2} converge, but at the point x_0, in correspondence with Eq. (20), A_c experiences a discontinuity. After the transition, A_C takes on the previous value, while the frequency of the self-oscillations remains equal to $4.36 \cdot 10^{-2}$ sec^{-1}.

NOTATION

ω, angular frequency; k, wave number; a, thermal diffusivity; T_T, thermostat temperature; T_C, transition temperature; T_0, the difference between the transition temperature and the thermostat temperature; T(x), the deviation of the temperature at point x from T_T; x, coordinate; t, real time; τ, normalized time; λ, thermal conductivity; ρ, density; u_1 and u_2, voltages; P, power; S, area; q, specific heat of transition; u_0, reference voltage; α, thermo-electromotive force coefficient; σ, Heaviside function; x_0, coordinate of the thermocouple; x_B, coordinate of the phase boundary (stationary component); $\tilde{x}_B$, the oscillating component of the phase boundary coordinate; ε, the small deviations of the parameters from their critical values; K, the controller gain coefficient; ξ, the dimensionless amplitude of the first harmonic; c_n and b_n, the Lyapunov coefficients; Θ_n and Ξ_n, the n-th harmonics of the temperatures of phases I and II, respectively; δ, thickness of the sample; x_n, the n-th harmonic of $\tilde{x}_B$; A, the generalized gain coefficient; B, the stationary temperature gradient; V(x) and W(x), the spatial parts of the first harmonic of the temperatures in phases I and II; $\varkappa$, φ_j, ψ_j, ℓ_j, μ_j, and D are auxiliary parameters.

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GENERALIZED STEFAN PROBLEM

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A generalized Stefan problem is considered in which volume heat release during the freezing-out of bound moisture is taken into account. It is shown that the appearance of additional criteria does not prevent obtaining a self-similar solution.

A whole series of problems associated with a change in the aggregate state of a material (freezing, drying, heating, sublimation, and similar problems) can be solved in terms of Stefan model approximations. In accordance with this model the phase separation boundary moves from the periphery into the depth of an object depending on withdrawal of heat from its surface (or the addition of heat to it). It is assumed here that the liberation or absorption of heating during a phase change takes place in an infinitely thin region of the material, namely, on a moving "front" (the phase separation boundary).

Experimental verification of the "frontal" theory yields satisfactory results in those cases involving moisture found in a free state. The situation deteriorates substantially when it becomes necessary to take the effect of bound moisture into account. We consider

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